FOR CLASS X 20-21 – MATHEMTICS – TEACHER MANISHA ROY

TOPIC – LINEAR EQUATIONS IN TWO VARIABLES REF : NCERT BOOK

1st Day (10.4.2020)

GOOD MORNING AND HAPPY GOOD FRIDAY TO MY DEAR STUDENTS!

chapter – 3 · Pair of Linear Equations.

I. General Form : $a_1x + b_1y + c_1 = 0$. —— (1)

and $a_2x + b_2y + c_2 = 0$. —— (2).

are two (Pair) of linear equations with two variables x and y [$a_1 \neq 0$, $b_1 \neq 0$, $a_2 \neq 0$, $b_2 \neq 0$].

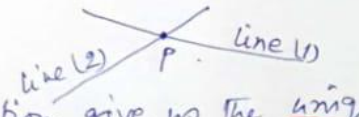
II. Equation (1) and (2) represent two Straight lines if drawn graphically on a graph paper.

III If we calculate the following ratio.

$\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$, $\frac{c_1}{c_2}$ the following result we will attained

A. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ [lines (1) & (2) will be intersecting

[CONSISTENT EQuation]

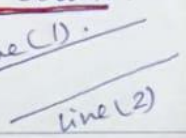


P being the point of intersection give us the unique solution of (1) and (2).

B If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ [lines (1) and (2) will be parallel and the lines will never intersect and will have no solution

[INCONSISTENT EQuation]

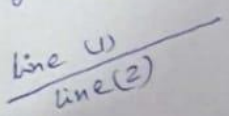
AS there will be no point of intersection there will be NO SOLUTION for (1) and (2).



C If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ [lines (1) and (2) will be overlapping – So we will get only one line

[CONSISTENT EQuation]

AS there will be only one line – so NO. of Solution will be infinite



Day-2 SATURDAY 11.4.2020

A. SOLUTION OF LINEAR EQUATIONS BY SUBSTITUTION METHOD

Ex. 3.3 (ii) Page: 53 NCERT

$$s - t = 3 \quad \text{---(1)}$$
$$\frac{s}{3} + \frac{t}{2} = 6 \quad \text{---(2)}.$$

From (1) $s = 3 + t \Rightarrow$ Substitute the vale of s in (2)

Putting in (2) $\frac{3+t}{3} + \frac{t}{2} = 6$

$$\frac{2(3+t)+3t}{6} = 6.$$

$$\frac{6+2t+3t}{6} = 6$$

$$\frac{6+5t}{6} = 6$$

$$6 + 5t = 36 \Rightarrow 5t = 36 - 6 = 30$$

$$t = \frac{30}{5} = 6.$$

$\therefore$ $s = 3 + t \Rightarrow s = 3 + 6 = 9$

Ans: $s = 9, \; t = 6.$

B. SOLUTION OF LINEAR EQUATIONS BY ELIMINATION METHOD.

Ex: 3.4 Page: 56

$$\frac{x}{2} + \frac{2y}{3} = -1 \quad \text{---(1)}$$
$$x - \frac{y}{3} = 3 \quad \text{---(2)}.$$

To eliminate x, multiply (2) by $\frac{1}{2} \Rightarrow \frac{x}{2} - \frac{y}{6} = \frac{3}{2}$ ---(3).

$$\frac{x}{2} + \frac{2y}{3} = -1 \quad \text{(1)}.$$
$$\frac{x}{2} - \frac{y}{6} = \frac{3}{2} \quad \text{(3)}.$$

$$\frac{2y}{3} + \frac{y}{6} = -1 - \frac{3}{2}$$

$$\frac{4y + y}{6} = \frac{-2-3}{2}$$

$$\frac{5y}{6} = -\frac{5}{2} \Rightarrow y = -\frac{5}{2} \times \frac{6}{5}^{3}$$

$$y = -3.$$

Put $y = -3$ in eqn (2)

$$x - \frac{-3}{3} = 3 \Rightarrow x + 1 = 3 \Rightarrow x = 2$$

Ans: $x = 2, \; y = -3.$

Day - 3 (SUNDAY) 12.4.2020

CROSS MULTIPLICATION METHOD

$$a_1x + b_1y + c_1 = 0 \quad \text{---(1)}$$
$$a_2x + b_2y + c_2 = 0. \quad \text{---(2)}$$

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

$$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

$a_1b_2 - a_2b_1 \neq 0$.
Provided $a_1b_2 \neq a_2b_1$
$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Case I: $a_1b_2 - a_2b_1 \neq 0$, unique solution.

Case II: $a_1b_2 - a_2b_1 = 0$. $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = k$

$\Rightarrow a_1 = a_2k, \; b_1 = b_2k$

From equation (1) We get

$$k(a_2x + b_2y) + c_1 = 0. \quad \text{---(3)}$$

Equation (3) and (2) can both be satisfied only if

$c_1 = kc_2$ i.e. $\frac{c_1}{c_2} = k$

If $c_1 = kc_2$ any solution of (2) will satisfy (1).
and vice versa.

So if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then there will be infinitely many solutions of (1) and (2).

Ex. 3.3

1 (v)

$$\sqrt{2}x + \sqrt{3}y = 0 \quad \text{---(1)}$$
$$\sqrt{3}x - \sqrt{8}y = 0 \quad \text{---(2)}$$

From (1) $\sqrt{2}x = -\sqrt{3}y$

$$x = -\frac{\sqrt{3}}{\sqrt{2}}y \quad \text{---(3)}$$

Substituting in (2)

$$\sqrt{3} \times \left(\frac{-\sqrt{3}}{\sqrt{2}}y\right) - \sqrt{8}y = 0$$

$$-\frac{3y}{\sqrt{2}} - \sqrt{8}y = 0$$

$$y\left(\frac{-3}{\sqrt{2}} - \sqrt{8}\right) = 0$$

$$y = \frac{0}{-\frac{\sqrt{3}}{2} - \sqrt{8}} = 0$$

From (3)

$$\therefore x = -\frac{\sqrt{3}}{\sqrt{2}}y = -\frac{\sqrt{3}}{\sqrt{2}} \times 0 = 0.$$

Ans: $x = 0$, $y = 0$.

3.4

1 (iv)

$$\frac{x}{2} + \frac{2y}{3} = -1 \quad \text{---(1)}$$

$$x - \frac{y}{3} = 3 \quad \text{---(2)}$$

(2) $\times \frac{1}{2}$ gives

$$\frac{x}{2} - \frac{y}{6} = \frac{3}{2} \quad \text{---(3)}$$

$$\frac{x}{2} + \frac{2y}{3} = -1 \quad \text{---(1)}$$

$$-\frac{y}{6} - \frac{2y}{3} = \frac{3}{2} + 1$$

$$\frac{-y - 4y}{6} = \frac{3+2}{2}$$

$$-\frac{5y}{6} = \frac{5}{2}$$

$$y = \frac{5}{2} \times -\frac{6}{5}\,3$$

$$y = -3$$

From (2)

$$\therefore \quad x = 3 + \frac{y}{3} = 3 + \frac{-3}{3} = 3 - 1 = 2.$$

Ans : $x = 2, \; y = -3.$

Ex. 3.5

4 (iii) Let Yash answered x no. of rightly and y no wrongly

$\therefore$ $3x - y = 40$ ——— (1)

$4x - 2y = 50$ ——— (2)

(1) $\times 2 \Rightarrow$ $6x - 2y = 80$ ——— (3)

$4x - 2y = 50$ ——— (2)

$-\quad +\quad -$

$2x = 30 \Rightarrow x = \frac{30}{2} = 15$

From (1) $y = 3x - 40 = 3 \times 15 - 40 = 45 - 40 = 5$

Ans $\therefore$ Total no. of questions $x + y = 15 + 5 = 20$

Ex. 3.6

1 (i) Let $\frac{1}{\sqrt{x}} = u$, $\frac{1}{\sqrt{y}} = v$

$2u + 3v = 2$ ——— (1)

$4u - 9v = -1$ ——— (2)

(1) $\times$ (2) $4u + 6v = 4$ ——— (3)

$4u - 9v = -1$ ——— (2)

$+\quad +$

$15v = 5 \Rightarrow v = \frac{5}{15} = \frac{1}{3}$

From (1) $2u = 2 - 3v = 2 - 3 \times \frac{1}{3} = 2 - 1 = 1$

$2u = 1 \Rightarrow u = \frac{1}{2}$

$\therefore \frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$ And $\frac{1}{\sqrt{y}} = \frac{1}{3}$

$\sqrt{y} = 3 \Rightarrow y = 9$

EX. 3.6

1 (v) $\frac{7x}{xy} - \frac{2y}{xy} = 5$

$\frac{7}{y} - \frac{2}{x} = 5$

$\frac{8}{y} + \frac{7}{x} = 15$

Let $\frac{1}{y} = u$, $\frac{1}{x} = v$.

$7u - 2v = 5$ —— (1)

$8u + 7v = 15$ —— (2).

105
40
65

(1) × 8 $56u - 16v = 40$

(2) × 7 $56u + 49v = 105$

$-65v = -65 \Rightarrow v = 1$

From 1) $7 \times u - 2 \times 1 = 5 \Rightarrow 7u = 5 + 2 = 7$

$u = 1$

$v = 1$, $u = 1$

$\Rightarrow x = 1$, $y = 1$. Ans.

1 (viii) $\frac{1}{3x+y} = u$, $\frac{1}{3x-y} = v$.

$$u + v = \frac{3}{4} \quad \text{--- (1)}$$

$$\frac{1}{2}u - \frac{1}{2}v = -\frac{1}{8} \quad \text{--- (2)}.$$

(1) $\times \frac{1}{2} \Rightarrow$ $\frac{1}{2}u + \frac{1}{2}v = \frac{3}{8}$

$\frac{1}{2}u - \frac{1}{2}v = -\frac{1}{8}$

$+ \quad +$

$$v = \frac{3}{8} + \frac{1}{8} = \frac{3+1}{8} = \frac{4}{8} = \frac{1}{2}$$

$v = \frac{1}{2}$, u =

From (1) $u = \frac{3}{4} - v = \frac{3}{4} - \frac{1}{2} = \frac{3-2}{4} = \frac{1}{4}$

$u = \frac{1}{4}$.

$\therefore \frac{1}{3x+y} = \frac{1}{4} \Rightarrow 3x+y = 4$ --- (3)

and $\frac{1}{3x-y} = \frac{1}{2} \Rightarrow 3x-y = 2$ --- (4).

Adding $6x = 6$

$x = \frac{6}{6} = 1$

From (3) $\therefore y = 4 - 3x = 4 - 3 = 1$

$\therefore$ Ans. $x = 1, y = 1$

ASSIGNMENT 1
LINEAR EQUATIONS IN 2 VARIABLES

1. Which of the following is not a solution of the pair of equations $3x - 2y = 4$ and $6x - 4y = 8$
   (a) $x = 2, y = 1$ (b) $x = 4, y = 4$
   (c) $x = 6, y = 7$ (d) $x = 5, y = 3$

2. The graphical representation of the pair of equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ provides us a pair of
   (a) Parallel lines (b) Intersecting lines
   (c) Coincident lines (d) None of these

3. The solution of the pair of equations $2x + 4y = 10$ and $3x + 5y = 12$ is
   (a) $x = 2, y = 1$ (b) $x = -1, y = 4$
   (c) $x = 2, y = -2$ (d) no solution

4. The value of K for which the pair of equations $kx - y = 2$ and $6x - 2y = 3$ has a unique solution is
   (a) $k = 3$ (b) $k = 3$
   (c) $k \neq 0$ (d) $k = 0$

5. Which of the following pairs of equations is inconsistent
   (a) $3x - 2y = 8$
   $2x + 3y = 1$
   (b) $3x - y = 8$
   $x - \frac{y}{3} = 3$
   (c) $x - y = 1$
   $x + y = 1$
   (d) $5x - y = 10$
   $10x - 2y = 20$

6. The vertices of the triangle obtained by the straight lines representing the linear equations $y = x$, $y = 0$, $x + y = 10$ are
   (a) (0, 0) (5, 5) (0, 10) (b) (0, 0) (5, 5) (10, 0)
   (c) (0, 0) (1, 1) (5, 5) (d) (0, 1) (1, 0) (1, 1)

7. The pair of linear equations $ax + by + c = 0$ and $px + qy + r = 0$ will represent parallel lines if
   (a) $\frac{a}{p} \neq \frac{b}{q}$ (b) $\frac{a}{p} = \frac{b}{q}$

(c) $\frac{a}{p} = \frac{b}{q} \neq \frac{c}{r}$ (d) $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$

8. If the pair of linear equations $3x + y = 1$ and $(2k - 1)x + (k - 1)y = 2k + 1$ is inconsistent, then the value of k is

(a) 1 (b) 0

(c) -1 (d) 2

9. The solution of the pair of equations $\frac{a}{x} - \frac{b}{y} = 0$ and $\frac{ab^2}{x} + \frac{a^2b}{y} = a^2 + b^2$ is

(a) x = a, y = b (b) x = -a, y = b

(c) x = a, y = -b (d) x = -a, y = -b

10. Five years ago, Namita was thrice as old as Mamta. Ten years later, Namita will be twice as old as Mamta. How old is Mamta at present?

(a) 50 years (b) 20 years

(c) 30 years (d) 15 years

ASSIGNMENT – 2
LINEAR EQUATIONS IN 2 VARIABLES

1. Determine the value of k so that the following pairs of equations represent coincident lines.
   (a) $x + 2y + 7 = 0;\ 2x + ky + 14 = 0$
   (b) $x + 5y - 7 = 0;\ 4x + 20y + k = 0$

2. Determine the value of k so that the following pairs of equations are inconsistent.
   (a) $(3k + 1)\,x + 3y - 2 = 0;\ (k^2 + 1)x + (k - 2)y - 5 = 0$
   (b) $kx + 3y = k - 2;\ 12x + ky = k$

3. Ramesh travels 760 km to his home partly by train and partly by car. He takes 8 hours if he travels 160 km by train and the rest by car. He takes 12 minutes more if he travels 240 km by train and the rest by car. Find the speed of the train and the car separately.

4. Two years ago, the father was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son. Find their present ages.

5. A 2-digit number is seven times the sum of its digits. The number formed by reversing the digits is 18 less than the original number. Find the number.

6. Some amount is distributed equally among some students. If there are 8 students less, every one will get Rs. 10 more. If there are 16 students more, every one will get Rs. 10 less. What is the number of students ? How much does each get ? What is the total amount distributed ?

7. 4 men and 4 boys can do a piece of work in 3 days, while 2 men and 5 boys can finish it in 4 days. How long would it take 1 boy to do it ? How long would it take 1 man to do it ?

8. A sailor goes 8 km down stream in 40 minutes and returns in 1 hour. Determine the speed of the sailor in still water and the speed of the current.

9. Points A and B are 90 Km apart from each other on a highway. A car starts from A and another from B at the same time. If they go in the same direction, they meet in 9 hours and if they go in the opposite directions they meet in 9/7 hours. Find the speeds of the two cars.

10. Solve for x and y

(a) $6x - 2y = 5xy,\ 2x + 6y = 5xy$

(b) $99x + 101y = 499;\ 101x + 99y = 501$

(c) $$\frac{1}{2(x+2y)} + \frac{5}{3(3x-2y)} = \frac{-3}{2}$$

$$\frac{-5}{4(x+2y)} - \frac{3}{5(3x-2y)} = \frac{61}{60}$$

(d) $$\frac{5}{x-1} + \frac{1}{y-2} = 2; \frac{6}{x-1} - \frac{3}{y-2} = 1$$

ANSWER

1. (a) $k = 4$
   (b) $k = -28$

2. (a) $k = -1$
   (b) $k = \pm 6$

3. 80 k / h, 100 k / h

4. 10 yrs. 42 yrs.

5. 42

6. 32 students, Rs. 30, Rs. 960

7. boy = 36 days, man = 18 days

8. Sailor – 10 k / h, speed of current = 2 k/h

9. A – 40 k / h, B – 30 k / h

10. (a) $x = 2, y = 1$
    (b) $x = 3,\ y = 2$
    (c) $x = \frac{1}{2}, y = \frac{5}{4}$
    (d) $x = 4,\ y = 5$